Children's Understanding of Luck

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This paper presents an analysis of two questionnaire items which explore students' understanding of the concept of luck in relation to the development of ideas of formal probability. The items were administered to 1014 students in Grades 3, 6 and 9 in Tasmanian schools. The analysis was based on the multimodal functioning SOLO The results lead to a model. hypothesised structure and implications for curriculum and teaching practice.

There has been debate about the development of children's understanding of probabilistic concepts since Piaget claimed that children needed to have access to formal thought before they could successfully solve problems in the field (e.g., Fischbein & Gazit, 1984; Hawkins & Kapadia, 1984). Other researchers have claimed that probabilistic reasoning is possible before the onset of formal operations and that the context in which problems are posed influences the responses obtained (e.g., Fischbein, 1975). More recently there has been a renewed interest in informal probability since the introduction of the Chance and Data component of the mathematics curriculum in A National Statement on Mathematics for Australian Schools (Australian Education Council [AEC], 1991) and Mathematics - A Curriculum Profile (AEC, 1994). The requirement to give children experiences related to chance from the early years of school opens the door to a consideration of children's understanding of luck. Luck, as a concept which can be considered intuitively as well as probabilistically, forms both a transition to more formal ideas and an adjunct to them once developed.

The two items analysed in this study were among 20 items in a paper-and-

pencil, short answer/multiple choice questionnaire devised to assess students' understanding of probability and statistics in Grades 3, 6 and 9 (Watson, Collis & Moritz, 1994). The items were selected for analysis because they each dealt with events allegedly involving luck, but in different contexts. The items, adapted from Fischbein and Gazit (1984), are presented in Figure 1. Item 1 requires the understanding that a child's perception of luck in the context of rising from bed in the morning is in fact pure superstition with no causal link. Item 2 presents information related to the luck of winning Tattslotto, requiring an appreciation that while a winner of Tattslotto has indeed had a good outcome in the past (has been lucky), all combinations of numbers still have the same chance of coming up in following draws.

1. Every morning, James gets out on the left side of the bed.

He says that this increases his chance of getting good marks.

What do you think?

2. One day, Claire won Tattslotto with the numbers 1; 7; 13; 21; 22; 36. So she said she would always play the same group of numbers, because they were lucky. What do you think about this?

Figure 1. Items 1 and 2.

The theoretical model chosen to describe the sophistication of various response types is the SOLO model based on the taxonomy developed by Biggs and Collis (1982) and later extended by them (Biggs & Collis, 1991; Collis & Biggs, 1991). This neo-Piagetian model incorporates multimodal functioning, which acknowledges the continued development of earlier modes of functioning in association with later modes. Of interest here are the ikonic and concrete symbolic modes, these being associated respectively with intuitive functioning and with the symbolic learning which takes place in school based on concrete materials. An additional possibility in the context investigated here is that an interaction between ikonic and concrete symbolic modes may be detected (e.g., Watson, Campbell & Collis, 1993).

Within modes learning cycles can be identified. The four levels of response for the learning cycle in this study, in increasing order of complexity are briefly:

- (i) <u>Prestructural responses (P)</u>, which represent no use of the elements required to identify the mode in question;
- (ii) <u>Unistructural responses (U)</u>, which represent the use of only one relevant aspect of the mode;
- (iii) <u>Multistructural responses (M)</u>, in which several disjoint relevant aspects are processed, usually in sequence; and
- (iv) <u>Relational responses (R)</u>, in which an integrated understanding of the relationships between the different aspects is achieved.

In this study Prestructural responses are equated with responses of the Ikonic (IK) mode.

It would be reasonable to expect that the items considered in this study would provoke ikonic functioning associated with intuitions and beliefs related to luck. As well as allowing for ikonic responses, the items would appear to require increasingly higher levels of concrete symbolic functioning to obtain correct solutions as judged by the school mathematics curriculum. A hypothesis for the structure associated with this increasingly sophisticated understanding will be given after the data are presented.

Sample and Methodology

Following the piloting of items (Watson, Collis & Moritz, 1994), eight regions of Tasmania were chosen for sampling. Cities, regional centres, and country regions were represented in the sample. Altogether 322 Grade 3, 310 Grade 6, and 382 Grade 9 students answered the 20-item questionnaire.

Questionnaires were administered during class time, which in high schools meant mathematics classes. In Grade 9 all students in Tasmania are required to enrol in mathematics. In high schools where classes were streamed, the classes chosen were the top or second top at Grade 9. In Grade 3 classes, the teacher assisted the researchers in helping children read the questions if required but no explanations were provided. All students were told that some questions may be very easy and some more difficult because the questions were being asked to students of different ages.

All responses were entered into a spreadsheet and then analysed using the language analysis software NUD•IST (Qualitative Solutions & Research, 1992). NUD•IST was programmed to index responses at nodes in a tree data structure by searching for words and character patterns. A search of this index system allowed logical operations on nodes; for example, the intersection of the index nodes [was|were] and [luck | lucky | fluke] resulted in an index of the responses which argued the result had been uncertain yet favourable in the past. By this method, the response set was partitioned into disjoint categories. The categories of response were then allocated SOLO modes and levels as outlined below. Following the allocation of levels, the percentages of response at each grade level were calculated to monitor changes over the three age It would be expected that groups. percentage changes would reflect more sophisticated thinking at higher grades. These results are presented graphically for easier interpretation.

Results

Item 1 - Left side of the bed

Of the 1014 students who attempted Item 1, 56 gave no response or an irrelevant one. The remaining 958 responses fell into five distinct categories.

There were students who responded with just Yes' (24 responses) or 'No' (100 responses). Simple Yes' or 'No' answers were not classified as they did not engage the question in a relevant propositional manner, nor did they provide a structure suitable for analysis using the SOLO model.

Responses which agreed with James that the side of the bed would help him (59 responses) were categorised as belonging to the ikonic mode, as they simply affirmed a belief in a superstition.

IK: 'I think he's right.'

IK: I get out the same side too. It's Bad luck getting out of the other side.'

Propositions which rejected the belief in a superstition were considered to show evidence of functioning in the concrete symbolic mode, but the reasoning associated with this varied in sophistication. Unistructural responses offered simple disagreement (591 responses).

- U: 'Not true.'
- U: 'It's just superstition.'
- U: 'It doesn't matter what side of the bed he gets out on. You still could fail or pass the test.'

At the next level, 107 multistructural responses added to the propositional reasoning by asserting that James' belief does not affect his chance of good marks, and by suggesting alternative concrete reasons for his marks at school.

- M: 'It won't help him get better marks. You only get good marks if you study and try hard.'
- M: 'I don't think he would unless there's a wall on the righthand side.'

At a higher level, 77 relational responses indicated that beliefs or psychological states concerning luck may influence physical outcomes which are under personal control. It is arguable that some of these responses are in transition to the next (or formal) mode as the students begin to introduce hypotheses from outside the data and use the given data to test them.

- R: I think he is wrong because whether he gets good marks or not depends on him, the test, whether or not he is concentrating, etc. Getting out the left side might possibly help a bit, though, because if he <u>thought</u> he would get good marks, he might.'
- R: 'It could if he thinks he could, and that would make him more confident in his work. Also it could affect the right and left side of the brain in some way.'

At all levels of the concrete symbolic mode there was evidence of ikonic support for some responses, as illustrated in the final response above.

A summary of these responses is given in Table 3 for comparison with Item 2. Table 1 shows the percentage of responses in each of the grades which occurred at each level, and Figure 2 shows this graphically.

Item 1	Grade 3		Grade 6		Grade 9		Total	
NR	18	6%	11	4%	3	1%	32	3%
Irrelevant	13	4%	9	3%	2	1%	24	2%
Yes only	21	7%	2	1%	1	0%	24	2%
No only	85	26%	15	5%	0	0%	100	10%
IK	31	10%	24	8%	4	1%	59	6%
U	137	43%	196	63%	258	68%	591	58%
M	14	4%	35	11%	58	15%	107	11%
R	3	1%	18	6%	56	15%	77	8%
Total	322	100%	310	100%	382	100%	1014	100%

Table 1. Item 1 responses by grade.



Figure 2. Item 1 SOLO levels by grade. Item 2 - Tattslotto numbers

In this item students were asked to comment on the Tattslotto numbers which Claire considers lucky. Of 1014 students, 53 gave no response or an irrelevant one; some students responded simply Yes' (13) or 'No' (55).

Responses were judged to be ikonic if they simply affirmed a (mystical) belief in being lucky (113 responses).

- IK: 'I think it would be lucky I will pick the same number's too.'
- IK: I don't think many numbers are lucky. But I think 4, 7 & 9 are, so I guess I'd agree in a way you can have lucky numbers.'

Unistructural responses were those which rejected the simple 'luck' belief, or which stated that the numbers were either unlikely to occur again or less likely to occur than other numbers (520 responses).

- U: 'There is no such thing as 'lucky numbers'.'
- U: 'I think she shouldn't go for the group of numbers again because you can't get the same numbers after numbers after numbers, you always get different numbers all the time.'

There were 218 responses which indicated an attempt to express the idea that all combinations of numbers have the same chance of occurring on any draw, and yet do not do so explicitly. These appeared to be in transition between unistructural and multistructural.

U-M: 'It was just a stroke of luck because any number could of come up.' U-M: 'There is no such thing as a lucky number, things like Tattslotto are picked at random.'

At the multistructural level (36 responses), often two or more relevant propositions were made, one of which was explicit in expressing the equality of chance for all numbers or combinations of numbers.

- M: 'She was fortunate for the numbers to come up but it is impossible for one number to be 'Luckier than another'.'
- M: 'There is equal chance for all combinations, but she's already won once, so why keep gambling, why not invest the money, you would get more out of it.'

Finally, six responses appealed to higher reasoning about psychological states as a reason for choosing certain numbers.

- R: It is a good idea to use the same numbers all the time but there is as much chance as getting any other six numbers.'
- R: I'd kill myself if I changed my numbers and the old sequence came up.'

A descriptive summary of responses is given in Table 3 for comparison with Item 1. Table 2 shows the percentage in each of the grades which occurred at each response level, and this is displayed in Figure 3.

Table 2. Item 2 responses by grade.								
Item 2	Grade 3		Grade 6		Grade 9		Total	
NR	34	11%	10	3%	9	2%	53	5%
Yes only	12	4%	0	0%	1	0%	13	1%
No only	48	15%	7	2%	0	0%	55	5%
IK	40	12%	42	14%	31	8%	113	11%
U	171	53%	168	54%	181	47%	520	51%
U-M	17	5%	80	26%	121	32%	218	21%
M	0	0%	3	1%	33	9%	36	4%
R	0	0%	0	-0%	6	2%	6	1%
Total	322	100%	310	100%	382	100%	1014	100%
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Figure 3. Item 2 SOLO levels by grade.

Hypothesised Structure for Understanding of Luck

Table 3 contains a brief summary of each category of response for the two items. The parallel nature of the responses at the various levels supports a SOLO-type structure of the development of understanding of luck in relation to chance. The association between responses to the two items was gauged by assigning integer values to levels and calculating a correlation coefficient. There was some degree of association (r_{792}) = .1776, p < .05), even though luck is set in different contexts in the two items. This association is shown in Table 4. Shaded cells indicate corresponding SOLO levels. The 118 responses which are unistructural for one item and higher level on the other are anomalous and perhaps can be explained by students giving functional rather than optimal answers to questions they consider trivial.

Table 3SOLO levels for responses to Items 1 and 2.

SOLO	Item 1. Lucky side of the bed.	Item 2. Lucky Tattslotto numbers.
-	No/Yes	No/Yes
Ikonic	Yes, it would help	Yes, they are lucky numbers
U	No, he is wrong	No, choose different numbers
U-M		She was just lucky, it's luck of the draw
М	No, it doesn't matter, you have to study	No, all numbers have the same chance
R	No, but more confident so works harder	No, same chance, but regret if change

Table 4 Number of responses by SOLO level for Items 1 and 2.

1 -				Item	2		
		IK	U	U- M	M	R	Tot al
	IK	18	26	4	1	1	50
ltem 1	U	57	338	141	26	1	563
	Μ	7	54	41	3	1	106
	R	6	37	23	6	3	75
	Tot al	88	455	209	36	6	794

Following previous work with fractions (Watson, Collis, & Campbell, in press), data handling (Watson, Collis, Callingham & Moritz, 1994), and the analysis in this study, the following structure for the development of the concept of luck is proposed, the U-M-R sequence being part of the concrete symbolic mode.

IK(P): Ikonic responses indicate belief that some outcomes may be explained by good or bad luck, and that future events may be predicted by luck.

U: At the unistructural level there is the recognition that although the outcome of a physical action may be uncertain, outcomes cannot be predicted simply on the basis of good or bad luck which forms part of a belief system.

M: At the multistructural level, there is admission of degrees of uncertainty, that events are more, less or equally likely. A sequence of alternative causal accounts of the event which might affect the balance of likelihood is usually provided.

R: At the relational level, responses indicate that likelihood of events is not accounted for by luck, but that some concrete integrating principles are involved. Responses may begin to hypothesise psychological reasons for behaviour, acknowledging that personal belief in luck may influence psychological states which in turn may influence physical outcomes.

It is hypothesised that ikonic reasoning decreases as students begin to rely on concrete symbolic reasoning to explain and qualify (or quantify) uncertain future outcomes. This is shown in Figures 2 and 3. Reflection upon personal and others' ikonic beliefs may lead to a more sophisticated integration of quasi-psychological and concrete symbolic reasoning which may be a mark of maturity in this area of chance.

Implications for Teaching and Curriculum Development

The concept of luck is one which permeates society for people of all ages. It is important for teachers and curriculum planners to be aware of the stages likely to occur in children's development in order to respond realistically and to encourage reasoning at higher levels where possible. For Item 1, some children have the egocentric perception that physical events are controlled by their thoughts or actions, which to most adults appears to be superstitious belief or the illusion of causal linkage. Others state definitively there is no causal mechanism here, while at a higher level, some students speculate that a psychological mechanism related to confidence may have some influence. For Item 2, some believe in lucky numbers and describe their own lucky numbers. Some students hold either positive or negative recency views, that the numbers are more likely or less likely to occur again following the recent outcome. Other students understand that all numbers, or all combinations of numbers, have the same chance, with some adding that sticking with the same numbers might save future regret in the event of the same combination occurring. Teachers need to be aware for example that a response to Item 2, such as 'No she's wrong', should be explored further to determine whether the child believes, 'the luck is used up' or 'all possible combinations of numbers have the same chance.' There is a great difference in the reasoning associated with the two explanations and comparing responses from class members may create the cognitive conflict to allow some children to move to more sophisticated levels in their understanding.

Curriculum planners also need to be more aware of the relationship of various beliefs about luck in preparing activities related to chance. Activities are needed that challenge the concept of luck which implies increased likelihood, and lead students to a concept which implies an appreciation of equal likelihood.

Acknowledgment

This research was funded by the Australian Research Council, Grant Number A79231392.

References

- Australian Education Council. (1991). A national statement on mathematics for Australian schools. Carlton, Vic. : Author.
- Australian Education Council. (1994). Mathematics -A curriculum profile for Australian schools. Carlton, Vic.: Curriculum Corporation.
- Biggs, J. B., & Collis, K. F. (1982). Evaluating the quality of learning: The SOLO taxonomy. New York: Academic Press.
- Biggs, J. B., & Collis, K. F. (1991). Multimodal learning and the quality of intelligent behaviour. In H. A. H. Rowe (Ed.), *Intelligence: Reconceptualisation and measurement* (pp. 57-76). Hillsdale, N. J.: Lawrence Erlbaum.
- Collis, K. F., & Biggs, J. B. (1991). Developmental determinants of qualitative aspects of school learning. In G. Evans (Ed.) *Learning and teaching cognitive skills* (pp. 185-207). Melbourne: Australian Council for Educational Research.
- Fischbein, E. (1975). The intuitive sources of probabilistic thinking in children. Dordrecht: D. Reidel.
- Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? An exploratory research study. *Educational Studies in Mathematics*, 15, 1-24.
- Hawkins, A., & Kapadia, R. (1984). Children's conception of probability - A psychological and pedagogical review. *Educational Studies in Mathematics*, 15, 349-377.
- Qualitative Solutions and Research. (1992). Nonnumerical Unstructured Data • Indexing Searching and Theorizing (NUD•IST) v3.0. [Computer program]. Melbourne: LaTrobe University.
- Watson, J. M., Campbell, K. J., & Collis, K. F. (1993). Multimodal functioning in understanding fractions. Journal of Mathematical Behaviour, 12, 45-62.
- Watson, J. M., Collis, K. F., Callingham, R. A., & Moritz, J. B. (1994). *Data cards - A pilot study of higher order thinking in statistics*. Paper presented at the Australian Association for Research in Education Conference, Newcastle, NSW.
- Watson, J. M., Collis, K. F., & Campbell, K. J. (in press). Developmental structure in the understanding of common and decimal fractions. *Focus on Learning Problems in Mathematics*.
- Watson, J. M., Collis, K. F., & Moritz, J. B. (1994).
 Assessing statistical understanding in grades 3, 6 and 9 using a short-answer questionnaire. In G.
 Bell, B. Wright, N. Leeson & J. Geake (Eds.), *Challenges in mathematics education: Constraints on construction* (pp. 675-682). Lismore, NSW: Mathematics Education Research Group of Australasia.